Introduction to Proofs - Introduction

Prof Mike Pawliuk

UTM

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Slides available at: mikepawliuk.ca

Learning Objectives (for this video)

By the end of this video, participants should be able to:

1. Prepare themselves for the course.
2. Describe the agony and the ecstasy of mathematics.

Course overview

- 12 weeks
- Textbook: "Introduction to Mathematical Proofs" by Shay Fuchs.
- Course Website: `https://q.utoronto.ca/courses/151962`
- Prerequisites: High school math (algebra mostly).

Learning Objectives (for this course)

By the end of this course, participants should be able to:

1. **Communicate mathematically**.
   1. Communicate mathematical ideas/arguments/proofs, using the language of mathematics, including terminology and notation, in an accurate, precise and coherent way.
   2. Parse a mathematical sentence about fundamental notions and objects (such as sets, functions, divisibility, relations), as well as their properties.
2. **Think mathematically**.
   1. Identify and implement methods of proof (e.g., contradiction, induction) that can be used to prove a given mathematical statement.
   2. Detect flaws and gaps in a mathematical argument, and identify ways to fix them.
   3. Adapt ideas and techniques in given proofs to solve new problems.

The agony and the ecstasy, part 1

Reflection

What feelings come to mind when you think about doing math?

Source: Scheinerman, Edward A. Mathematics: a discrete introduction. Nelson Education, 2012.

The agony and the ecstasy, part 2

Question 1

Simplify

$$(x-a)(x-b)(x-c)\cdots(x-z)$$

Source: Scheinerman, Edward A. Mathematics: a discrete introduction. Nelson Education, 2012.

The agony and the ecstasy, part 3

Question 2

There are 25 ants on a meter stick. Ants walk at 1 cm/s in the direction they are facing (either left or right). When two ants bump into each other, they bounce in opposite directions without losing any speed. If an ant reaches the edge, it walks off.
Is it possible for at least one ant to be on the meter stick after 120 seconds?

The agony and the ecstasy, part 4

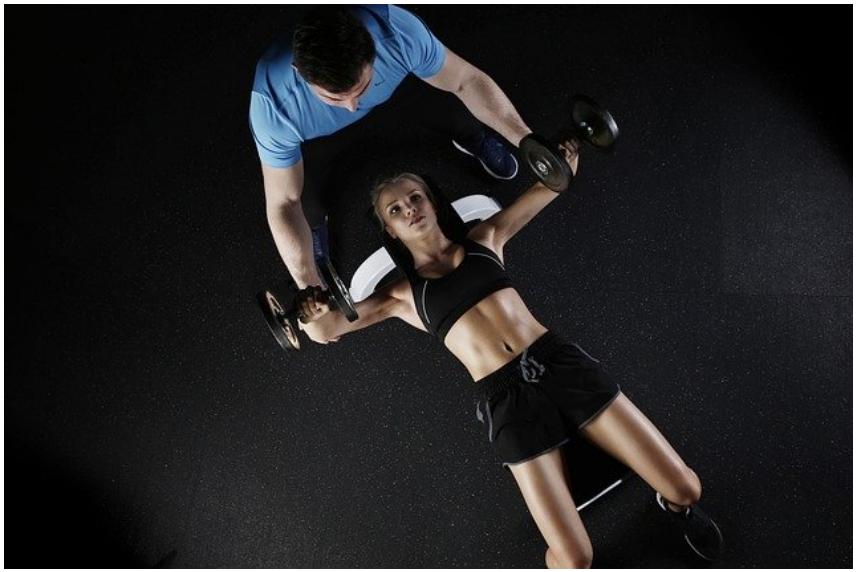
Warning!

Do not under any circumstances ask anyone for a solution. If you know a solution, do not share it with others.

Advice for succeeding in this course

Advice 1

Work on many problems.



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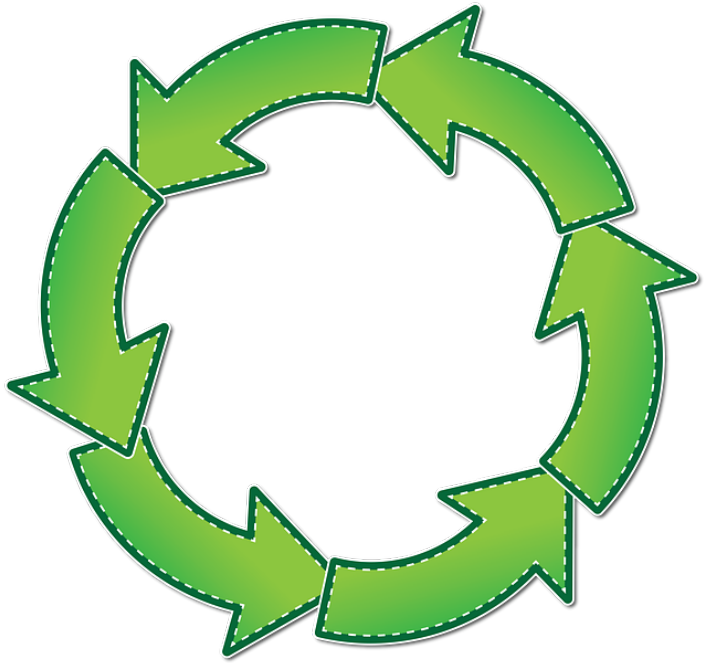
Advice for succeeding in this course

Advice 2

Get stuck and get unstuck.



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Advice for succeeding in this course

Advice 3

Ask lots of questions.

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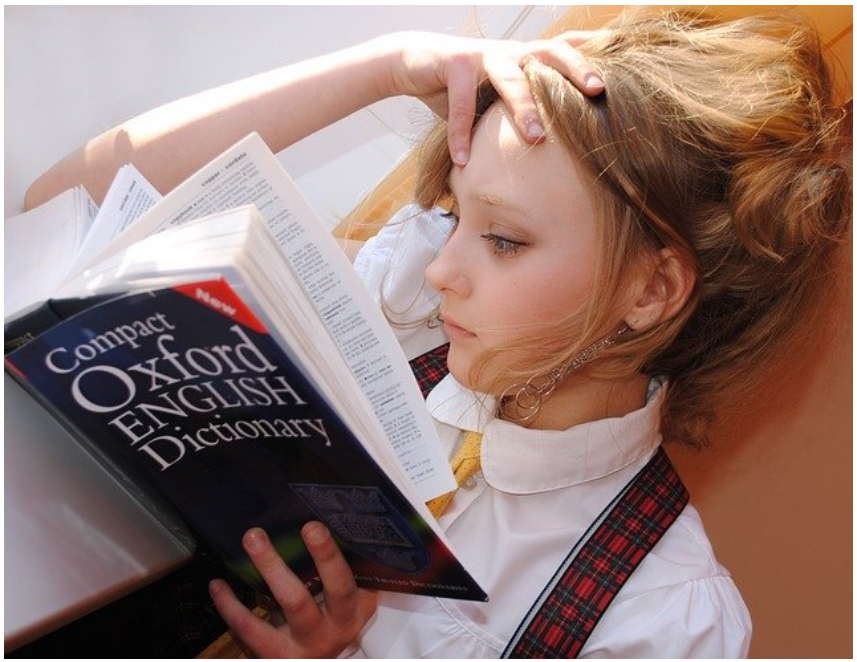
Advice for succeeding in this course

Advice 4

Learn the definitions, but don't memorize anything else.



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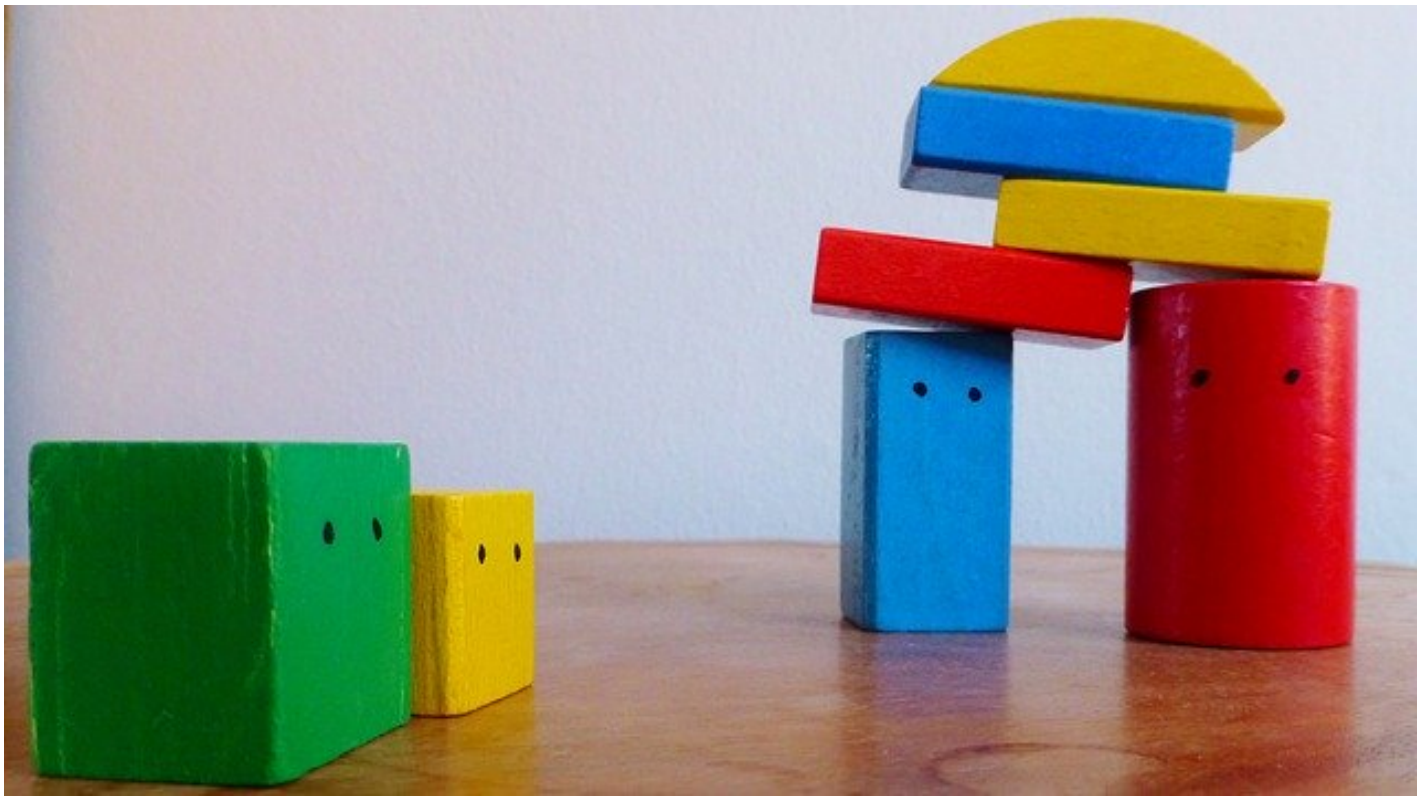
Advice for succeeding in this course

Advice 5

Play.



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Advice for succeeding in this course

Advice 6

Make lots of conjectures.

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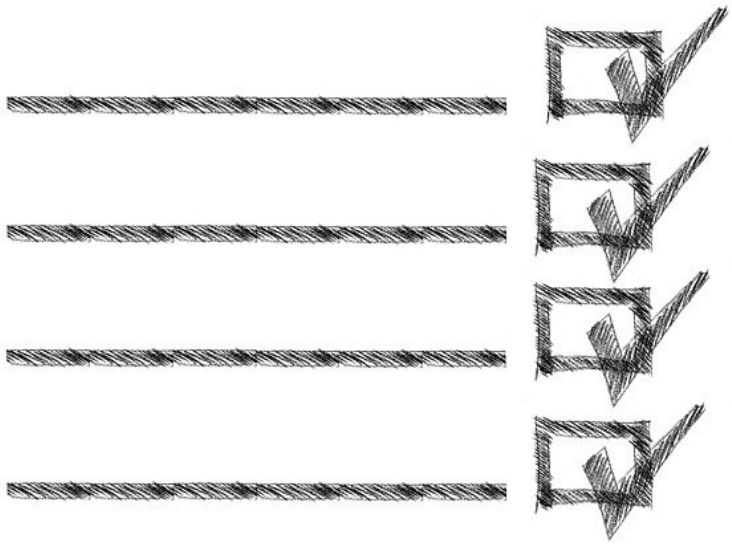
Advice for succeeding in this course

Advice 7

Prove **everything** you can.



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Advice for succeeding in this course

1. Work on many problems.
2. Get stuck and get unstuck.
3. Ask lots of questions.
4. Learn the definitions, but don't memorize anything else.
5. Play.
6. Make lots of conjectures.
7. Prove **everything** you can.

Reflection

- What are the two key learning objectives in this course?
- What is a piece of advice for succeeding in the course that you like/dislike?
- How will you follow it?