MAT102H5 F - FALL 2019 - PROBLEM SET 2

Submission

- **You must submit your completed problem set on Quercus by 5:00pm Friday October 4, 2019**.
- Late assignments will not be accepted.
- Consider submitting your assignment well before the deadline.
- If possible, please submit your assignment as a PDF, or as a clear picture.
- If you require additional space, please insert extra pages.

Additional Instructions

You must justify and support your solution to each question.

Academic Integrity

You are encouraged to work with your fellow students while working on questions from the problem sets. However, the writing of your assignment must be done without any assistance whatsoever.

I, ____, affirm that this assignment represents entirely my own
STUDENT NAME
efforts. I confirm that:

- I have not copied any portion of this work.
- I have not allowed someone else in the course to copy this work.
- This is the final version of my assignment and not a draft.
- I understand the consequences of violating the University's academic integrity policies as outlined in the *Code of Behaviour on Academic Matters*.

By signing this form I agree that the statements above are true.

If I do not agree with the statements above, I will not submit my assignment and will consult the course coordinator (Professor Mike Pawliuk) immediately.

Student Name: ____

Student Number: ____

Signature: ____

Date: ____

Problem 1. Let S be the collection of all statements of the form

$$P\square P\square P\square P\square P\square P\square P$$

where each $\square$ is either a '$\vee$' or a '$\wedge$'. (There are 6 squares, and $2^6 = 64$ such statements in S.) For example, the statement

$$P \vee P \vee P \wedge P \vee P \wedge P \wedge P$$

is in S. (Note that $\wedge$ is evaluated before $\vee$.)
Show that all statements in S are logically equivalent.

Problem 2. Let T be the collection of all statements of the form

$$P\square P\square P\square P\square P\square P\square P$$

where each $\square$ is either a '$\vee$' or a '$\wedge$', and there is EXACTLY ONE $\square$ that is a '$\Rightarrow$'. For example, the statement

$$P \vee P \vee P \wedge P \vee P \Rightarrow P \wedge P$$

is in T. (Note that $\wedge$ is evaluated before $\vee$, and $\vee$ is evaluated before $\Rightarrow$.)
Show that all statements in T are logically equivalent.

Problem 3 (You do not need to submit a solution to this question). Generalize the previous question to any number of '$\Rightarrow$' symbols.

Problem 4. (3.7.30 from textbook) Use contrapositive to prove this. Let $x, y \in \mathbb{R}$. If x and y are both positive, then $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$.

Problem 5. Use proof by contradiction to prove "if the sum of two prime numbers is prime, then one of the primes must be 2".

Problem 6. One of DeMorgan's laws (for logic), is that:

$$\neg(P \wedge Q) \text{ is logically equivalent to } (\neg P) \vee (\neg Q).$$

Use this to prove DeMorgan's first law for sets:

$$\text{If } A, B, C \text{ are sets, then } A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C).$$

Problem 7 (You do not need to submit a solution to this question). Are you able to prove DeMorgan's law for logic? If so how? Also, reflect on how similar looking the two versions of DeMorgan's laws are.

Problem 8. Create an example of a function $f : \mathbb{R} \to \mathbb{R}$ such that $f(f(f(\mathbb{R}))) = f(f(\mathbb{R})) \neq f(\mathbb{R})$.
(Note: I will give a hint for this question on Friday September 27 at 5:00pm. Until then, do not ask anyone for hints, and try to solve it on your own.)