MAT102H5 F - FALL 2019 - PROBLEM SET 3

Submission

- **You must submit your completed problem set on Quercus by 5:00pm Friday October 25, 2019**.
- Late assignments will not be accepted.
- Consider submitting your assignment well before the deadline.
- If possible, please submit your assignment as a PDF, or as a clear picture.
- If you require additional space, please insert extra pages.
- **You must include a signed and completed version of this cover page**.

Additional Instructions

You must justify and support your solution to each question.

Academic Integrity

You are encouraged to work with your fellow students while working on questions from the problem sets. However, the writing of your assignment must be done without any assistance whatsoever.

I, ____, affirm that this assignment represents entirely my own
STUDENT NAME
efforts. I confirm that:

- I have not copied any portion of this work.
- I have not allowed someone else in the course to copy this work.
- This is the final version of my assignment and not a draft.
- I understand the consequences of violating the University's academic integrity policies as outlined in the *Code of Behaviour on Academic Matters*.

By signing this form I agree that the statements above are true.

If I do not agree with the statements above, I will not submit my assignment and will consult the course coordinator (Professor Mike Pawliuk) immediately.

Student Name: ____

Student Number: ____

Signature: ____

Date: ____

Recall that two fractions $\frac{p}{q}, \frac{a}{b}$ are equivalent if $pb = qa$. For example, $\frac{3}{6}$ is equivalent to $\frac{4}{8}$ because $3 \cdot 8 = 24 = 6 \cdot 4$. Also, recall that a transversal is a set of elements such that no two elements are equivalent.

Consider the list:

$$\frac{1}{2}, \frac{2}{1}, \frac{0}{1}, \frac{10}{5}, \frac{5}{2}, \frac{2}{5}, \frac{0}{2}, \frac{1}{1}, \frac{-10}{-5}, \frac{5}{10}, \frac{4}{2}, \frac{-2}{5}$$

Problem 1. What is the number of elements of the largest transversal subset of that list? Briefly explain how you know your answer is correct.
Optional: If you are trying to make CS Post, this question should be too easy for you. Instead, solve the same question but with the list `ps3.txt`, posted on Quercus, which contains 1000 fractions. Briefly explain how you solved this question (but you do not need to submit any code).

Problem 2. What is the number of elements of the largest equivalence class that is a subset of that list? Briefly explain how you know your answer is correct.
Optional: If you are trying to make CS Post, this question should be too easy for you. Instead, solve the same question but with the list `ps3.txt`, posted on Quercus, which contains 1000 fractions. Briefly explain how you solved this question (but you do not need to submit any code).

Problem 3 (**You do not need to submit a solution to this question**)**.** The following question is a related open question (meaning no one has ever solved it): Suppose that R is a symmetric, reflexive relation on the set $X = \{1, 2, 3, \ldots, n\}$ (and $n \approx 1000$). Describe an efficient way to find the largest equivalence class formed only from pairs in R. For more information read the Wikipedia article "Clique problem".

Problem 4. (7.5.13 from textbook) On the set $\mathbb{N} \times \mathbb{N}$, define the following relation:

$$(a, b) \sim (c, d) \text{ if and only if } a + d = b + c.$$

(1) Show that this is an equivalence relation.
(2) Describe the equivalence class of $(1, 1)$.

Problem 5. (1.5.6.a from textbook) Solve the inequality $\frac{2}{x} > 3x$.

Problem 6. (1.5.18 from textbook) Show that for any two positive real numbers a, b:

$$\frac{a}{a+2b} + \frac{b}{b+2a} \geq \frac{1}{2}.$$

(Do not present a backwards "proof" that starts from the conclusion, and derives something true. Start from something true, and eventually conclude this inequality.)

Problem 7. Prove that $(\forall n \in \mathbb{N})(\forall x \in \mathbb{Z})[x^{2n}$ is congruent to 0 or 1 modulo 4].

Problem 8. Use the previous problem to prove that

$$-1 + 4x + x^2 + 8x^3 + x^4 = 0$$

has no integer solutions.