MAT102H5 F - FALL 2019 - PROBLEM SET 5

Submission

- **You must submit your completed problem set on Quercus by 5:00pm Friday November 29, 2019**.
- Late assignments will not be accepted.
- Consider submitting your assignment well before the deadline.
- If possible, please submit your assignment as a PDF, or as a clear picture.
- If you require additional space, please insert extra pages.
- **You must include a signed and completed version of this cover page**.

Additional Instructions

You must justify and support your solution to each question.

Academic Integrity

You are encouraged to work with your fellow students while working on questions from the problem sets. However, the writing of your assignment must be done without any assistance whatsoever.

I, ____, affirm that this assignment represents entirely my own
STUDENT NAME
efforts. I confirm that:

- I have not copied any portion of this work.
- I have not allowed someone else in the course to copy this work.
- This is the final version of my assignment and not a draft.
- I understand the consequences of violating the University's academic integrity policies as outlined in the *Code of Behaviour on Academic Matters.*

By signing this form I agree that the statements above are true.

If I do not agree with the statements above, I will not submit my assignment and will consult the course coordinator (Professor Mike Pawliuk) immediately.

Student Name: ____

Student Number: ____

Signature: ____

Date: ____

Problem 1. Let $A \subseteq \mathbb{R}$ be a countable set. Prove that $\mathbb{R} \setminus A$ is uncountable.

The next two problems together will prove "$A = B$ if and only if $\mathcal{P}(A) = \mathcal{P}(B)$".

Problem 2. Let A, B be sets. Prove that if $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$. Explain why we can conclude that if $A = B$, then $\mathcal{P}(A) = \mathcal{P}(B)$.

Problem 3. Let A, B be sets. Prove that if $\mathcal{P}(A) \subseteq \mathcal{P}(B)$, then $A \subseteq B$. Explain why we can conclude that if $\mathcal{P}(A) = \mathcal{P}(B)$, then $A = B$.

The next three problems are all related.

Problem 4. Let A be a set. Define B to be the collection of all functions $f : \{1\} \to A$. Prove that $|A| = |B|$ by constructing a bijection $F : A \to B$.

Problem 5. Let A be a set. Define C to be the collection of all functions $f : \{0, 1\} \to A$. Prove that $|A \times A| = |C|$ by constructing a bijection $F : A \times A \to C$.

Problem 6. Let A be a set. Define D to be the collection of all functions $f : A \to \{0, 1\}$. Prove that $|\mathcal{P}(A)| = |D|$ by constructing a bijection $F : \mathcal{P}(A) \to D$.

In the following two problems we will refer to Cantor's Diagonalization Argument (as presented in pages 124 and 125 of the textbook). For these two questions we will replace the definition of a_k in the textbook with:

$$a_k = \begin{cases} 1 & \text{if the } k\text{th digit of } f(k) \text{ is } 0 \\ 0 & \text{otherwise} \end{cases}$$

Problem 7. You are a consultant for a friend designing a new video-game. Every player in the game is assigned a unique ID which is a binary string with 3 digits. Currently only the following three IDs have been assigned to players: $000, 001$ and 010.
Apply Cantor's Diagonalization argument to get an ID for a 4th player that is different from the three IDs already used.

Problem 8. (Continuing Problem 7) Show that from the four IDs (the three originals and the new diagonalized one) you can choose three IDs I_1, I_2, I_3 so that they will generate a 5th new ID when Cantor's Diagonalization Argument is applied to I_1, I_2, and I_3.
By repeatedly doing this, show that you can generate all 8 possible IDs.

Problem 9 (You do not need to submit a solution to this question.). Show that if you had started with any three IDs in Problem 7, then you can still generate all 8 possible IDs.