THE UNIVERSITY OF TORONTO - MISSISSAUGA - Quiz 2 -
Version 1
MAT102H5F - Fall 2019 - LEC0101-LEC0107
Time: 45 minutes
Date: Thursday October 10, 2019. 7:10PM - 7:55PM.
Instructors: M. Tvalavadze, S. Fuchs, N. Askaripour, X. Wang, A. Burazin M. Pawliuk.
Aids: None.

Instructions:

- Do not write on the QR code at the top of each page.
- Only the **front** of each page will be graded. (You may use the **backs** of pages for rough work.)
- You may use Page 5 for any additional work you want graded. If you use this page, please indicate that on the original page containing the question.
- Do not remove any pages.
- Your answers to the multiple choice questions must be recorded on the final page of the booklet.
- The quiz is out of 20 points.

Instructions: There are five (5) multiple choice questions worth two (2) points each. Choose the answer that is most correct. Your answers must be recorded on the final page of the booklet.

MC1 (2pts) Express the following statement by correctly using the logic symbols: "Every rational number x has a non-zero integer n such that xn is an integer."

A. $(\forall x \in \mathbb{Q})(\forall n \in \mathbb{Z} \setminus \{0\})(\exists y \in \mathbb{Z})[xn = y]$

B. $(\forall x \in \mathbb{Q})(\exists n \in \mathbb{Z} \setminus \{0\})(\exists y \in \mathbb{Z})[xn = y]$

C. $(\forall x \in \mathbb{Q})(\exists n \in \mathbb{Z})(\exists y \in \mathbb{Z} \setminus \{0\})[xn = y]$

D. $(\forall x \in \mathbb{Q}) \wedge (\exists n \in \mathbb{Z} \setminus \{0\}) \wedge (\exists y \in \mathbb{Z} \setminus \{0\}) \wedge [xn = y]$

E. $(\forall x \in \mathbb{Q})[xn = y](n \in \mathbb{Z} \setminus \{0\}, y \in \mathbb{Z})$

MC2 (2pts) Let $\emptyset$ be the empty set. Let $A = \{\emptyset\}$. Which statement is <u>false</u>?

A. $\emptyset \in A$
B. $\emptyset \subseteq A$
C. $\emptyset \neq A$
D. $A \not\subseteq \emptyset$
E. $A \subseteq \emptyset$

MC3 (2pts) Yixuan conjectures that: "If p is a prime, then p^2 is not a prime number". Which of the following proof strategies will prove this statement is true?

A. Note that $p = 3$ is prime, but $p^2 = 9$ is not prime.
B. Assume that p is not prime, so p^2 must also be not prime.
C. Assume that p^2 is not prime, and conclude that p is prime.
D. Assume that p^2 is prime, and conclude that p is not prime.
E. Assume that p is prime, and conclude that p^2 is also prime.

MC4 (2pts) Let $f : [-2, 2) \to \mathbb{R}$ be defined by $f(x) = x^2$. Which sets provide a counterexample to "$f(A \cap B) = f(A) \cap f(B)$"?

A. $A = [-2, 0], B = [0, 2]$.
B. $A = [-2, 2), B = [-2, 2)$.
C. $A = \emptyset, B = \emptyset$.
D. $A = \{-1, 0\}, B = \{0, 1\}$
E. $A = \mathbb{Q}, B = \mathbb{R} \setminus \mathbb{Q}$.

MC5 (2pts) Is $(P \Rightarrow P) \Rightarrow P$ logically equivalent to $P \Rightarrow (P \Rightarrow P)$?

A. Yes, they are both tautologies.
B. Yes, they are both contradictions.
C. Yes, but they are neither tautologies or contradictions.
D. No, but $P \Rightarrow (P \Rightarrow P)$ is a tautology.
E. No, but $(P \Rightarrow P) \Rightarrow P$ is a tautology.

Instructions: There are two (2) long answer questions worth five (5) points each, with multiple parts. Provide a complete solution, with justification, in the space provided.

Q1.1 (3 POINTS)

Write the negation of "$(\forall x, y \in \mathbb{R})(\exists z \in \mathbb{R})[(x < z) \Rightarrow (y < z)]$".

Q1.2 (2 POINTS)

Use proof by contradiction to prove that there are no rational solutions to

$$x^3 + x + 1 = 0.$$

Q2.1 (2 POINTS)

Let A, B be sets. Use only logic symbols $(\vee, \wedge, \Rightarrow, \Leftrightarrow, \neg, \in, \notin)$ to express the following statement: "$x \in (A \setminus B) \cup (B \setminus A)$ if ..."

Q2.2 (3 POINTS)

Suppose that $A \subseteq B$ and $C \subseteq D$. Prove that $(A \cap C) \subseteq (B \cup D)$.

Instructions: You may use this page for any additional work you want graded. If you use this page, please indicate that on the original page containing the question.

[End of Quiz]