THE UNIVERSITY OF TORONTO - MISSISSAUGA - Quiz 3 - Version 1

MAT102H5F - Fall 2019 - LEC0101-LEC0107

Time: 45 minutes

Date: Thursday November 14, 2019. 7:10PM - 7:55PM.

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Aids: None.

Instructions:

- Do not write on the QR code at the top of each page.
- Only the **front** of each page will be graded. (You may use the **backs** of pages for rough work.)
- You may use Page 6 for any additional work you want graded. If you use this page, please indicate that on the original page containing the question.
- Do not remove any pages.
- Your answers to the multiple choice questions must be recorded on the final page of the booklet.
- The quiz is out of 20 points.

Instructions: There are five (5) multiple choice questions worth two (2) points each. Choose the answer that is most correct. Your answers must be recorded on the final page of the booklet.

MC1 (2pts) Ali conjectures that "For every natural number $\sum_{i=1}^{n} i \leq \prod_{i=1}^{n} i$."

Is his conjecture true?

A. Yes, because sums are always less than or equal to products.
B. Yes, because $i \leq i$ is always true.
C. No, because for $n = 4$ this is a strict inequality.
D. No, because the inequality is not true for $n = 1$.
E. No, because the inequality is not true for $n = 2$.

MC2 (2pts) Which of the following statements is a more general version of the observation that

$$1 + 4 + 7 + 10 + 13 = \frac{5(14)}{2}.$$

A. $\forall n \in \mathbb{N}, \sum_{i=1}^{n}(3i - 2) = \frac{n(3n-1)}{2}$.
B. $\forall n \in \mathbb{N}, \sum_{i=1}^{5}(3i - 2) = \frac{n(3n-1)}{2}$.
C. $\forall n \in \mathbb{N}, \sum_{i=1}^{n}(3i - 2) = \frac{i(3i-1)}{2}$.
D. $\sum_{i=1}^{13}(3i - 2) = \frac{5(14)}{2}$.
E. $\sum_{i=1}^{13} i = \frac{5(14)}{2}$.

MC3 (2pts) What is the range of the function $f : \mathbb{R}\setminus\{0\} \to \mathbb{R}$ defined by $f(x) = \dfrac{|x| + x}{x^2}$?

A. $(0, \infty)$
B. $[0, \infty)$
C. $(-\infty, \infty)$
D. $[1, \infty)$
E. $\mathbb{R} \setminus \{0\}$

MC4 (2pts) Pranav has a mathematical statement $P(n)$. Pranav knows:

(1) $P(n)$ is true for all prime numbers n, and
(2) Whenever $P(n)$ is true, then $P(n-1)$ is also true.

The set all of numbers n that Pranav knows $P(n)$ is true is:

A. $\mathbb{N}$.
B. The set of all prime numbers.
C. $\mathbb{Z}$
D. The set of all natural numbers $n \geq 2$.
E. $\emptyset$.

MC5 (2pts) Which of the following is the induction hypothesis for strong induction when used to prove "$\forall n \in \mathbb{N}, P(n)$".

A. $P(1)$.
B. $P(n)$ for all $n \in \mathbb{N}$.
C. $P(n)$ for a particular $n \in \mathbb{N}$.
D. $P(1), \ldots, P(n)$ for all $n \in \mathbb{N}$.
E. $P(1), \ldots, P(n)$ for a particular $n \in \mathbb{N}$.

Instructions: There are two (2) long answer questions worth five (5) points each, with multiple parts. Provide a complete solution, with justification, in the space provided.

Q1.1 (3 points)

Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \dfrac{2x}{1+x^2}$. Show that $\dfrac{1}{3}$ is an element of the range of f. (Do not use calculus.)

Q1.2 (2 points)

Prove that

$$\sum_{i=1}^{4} i^2 = \sum_{i=1}^{4} i + \sum_{i=2}^{4} i + \sum_{i=3}^{4} i + \sum_{i=4}^{4} i.$$

Recall that the Fibonacci numbers are defined as

$$F_1 = 1, F_2 = 1 \text{ and } F_{n+2} = F_{n+1} + F_n \text{ for all } n \in \mathbb{N}.$$

Q2.1 (2 POINTS)

Is F_{303} an even number? Circle one of the following. (No proof needed)

Yes **No**

Q2.2 (3 POINTS)

Prove that for all $n \in \mathbb{N}$ that $F_1 + F_2 + \ldots + F_n = -1 + F_{n+2}$.

Instructions: You may use this page for any additional work you want graded. If you use this page, please indicate that on the original page containing the question.

[End of Quiz]