THE UNIVERSITY OF TORONTO - MISSISSAUGA - Quiz 4 - Version 1
MAT102H5F - Fall 2019 - LEC0101-LEC0107
Time: 45 minutes
Date: Thursday November 21, 2019. 7:10PM - 7:55PM.
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Aids: None.

Instructions:

- Do not write on the QR code at the top of each page.
- You may use the **back** of the cover page, and pages 2-3 for rough work.
- You may use Page 6 for any additional work you want graded. If you use this page, please indicate that on the original page containing the question.
- Do not remove any pages.
- Your answers to the multiple choice questions must be recorded on the final page of the booklet.
- The quiz is out of 20 points.

Instructions: You may use this page for any additional work you want graded. If you use this page, please indicate that on the original page containing the question.

Instructions: There are five (5) multiple choice questions worth two (2) points each. Choose the answer that is most correct. Your answers must be recorded on the final page of the booklet.

MC1 (2pts) Let $f : A \to B$ and $g : C \to D$ be functions. What condition is necessary for $g \circ f$ to be defined?

A. $A = D$,
B. $B = C$,
C. $B \subseteq C$,
D. range $g \subseteq A$,
E. range $f \subseteq C$.

MC2 (2pts) Let M, N be natural numbers. What is:

$$\sum_{j=1}^{N} \left(\sum_{i=1}^{M} 1 \right)?$$

A. 1,
B. $M + N$,
C. $(M - 1)(N - 1)$,
D. MN,
E. $(M + 1)(N + 1)$.

MC3 (2pts) How many functions $f : \{1, 2, \ldots, 2019\} \to \{0, 1\}$ are there that are <u>not</u> surjective?

A. 0,
B. 1,
C. 2,
D. 2019,
E. 2^{2019}.

MC4 (2pts) The definition of a dyadic number is given recursively as follows:

(1) 0 is dyadic,

(2) 1 is dyadic, and

(3) Whenever a is dyadic and b is dyadic, then $\dfrac{a+b}{2}$ is dyadic.

Which of the following numbers is dyadic?

A. $\dfrac{1}{8}$,

B. $\dfrac{1}{\sqrt{2}}$,

C. $\dfrac{1}{6}$,

D. $\dfrac{1}{3}$,

E. $\dfrac{1}{5}$.

MC5 (2pts) Let $f : \{-1, 0, 2\} \to \mathbb{R}$ be defined by $f(x) = x^2$. Is this function an injection?

A. No, because x^2 fails the vertical line test.

B. No, because $(1)^2 = (-1)^2$ and $-1 \neq 1$.

C. No, because $0^2 = 0$.

D. Yes, because x^2 passes the vertical line test.

E. Yes, because if $f(a) = f(b)$, then $a = b$.

Instructions: There are two (2) long answer questions worth five (5) points each, with multiple parts. Provide a complete solution, with justification, in the space provided.

Recall that the Fibonacci numbers are defined by $F_1 = 1, F_2 = 2$ and $F_{n+2} = F_{n+1} + F_n$ for all $n \in \mathbb{N}$.

Q1.1 (2 points)

Prove (directly, without induction) that $F_{n+4} = 3F_{n+1} + 2F_n$, for every $n \in \mathbb{N}$.

Q1.2 (3 points)

Prove by induction that for every natural number n, that F_{4n} is a multiple of 3. (You may use the result of Q1.1.)

Q2.1 (2 points)

A function $f : \mathbb{R} \to \mathbb{R}$ is defined to be bounded above if

$$(\exists M \in \mathbb{R})(\forall x \in \mathbb{R})[f(x) \leq M].$$

Prove that if $f : \mathbb{R} \to \mathbb{R}$ is bounded above, then f is not a surjection.

Q2.2 (3 points)

Let $f : A \to B$ be a bijection and $g : B \to A$ be a function such that $f \circ g \circ f(a) = f(a)$ for all $a \in A$. Prove that $g = f^{-1}$.

Instructions: You may use this page for any additional work you want graded. If you use this page, please indicate that on the original page containing the question.

[End of Quiz]