MAT102H5 Y - SUMMER 2020 - PROBLEM SET 1

Submission

- **You must submit your completed problem set on Crowdmark by 5:00 pm (EDT) Friday May 15, 2020**.
- Late assignments will not be accepted.
- Consider submitting your assignment well before the deadline.
- If you require additional space, please insert extra pages.
- You do not need to print out this assignment; you may submit clear pictures/scans of your work on lined paper, or screenshots of your work.
- **You must include a signed and completed version of this cover page**.

Additional Instructions

You must justify and support your solution to each question.

Academic Integrity

You are encouraged to work with your fellow students while working on questions from the problem sets. However, the writing of your assignment must be done without any assistance whatsoever.

By signing this statement I affirm that this assignment represents entirely my own efforts. I confirm that:

- I have not copied any portion of this work.
- I have not allowed someone else in the course to copy this work.
- This is the final version of my assignment and not a draft.
- I understand the consequences of violating the University's academic integrity policies as outlined in the *Code of Behaviour on Academic Matters.*

By signing this form I agree that the statements above are true.
If I do not agree with the statements above, I will not submit my assignment and will consult my course instructor immediately.

Student Name: ____

Student Number: ____

Signature: ____

Date: ____

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Problem 1. The syllabus for MAT102 says:

(P_1) Your "Best 5 (out of 6) problem sets" will make your problem set mark.

(P_2) "Your problem set mark will be determined by taking the average of the best five problem sets.".

(1) Compute P_1 and P_2 for the problem set grades: $10/15, 14/16, 1/10, 9/16$, [did not hand in PS5], $20/20$.

(2) Explain why (i.e. prove) no matter what 6 grades a student received on their problem sets, $P_1 = P_2$. (**Suggestion**: Your proof should start with "Let $x_1, x_2, x_3, x_4, x_5, x_6$ be the grades a student has received on their (respective) problem sets.")

Problem 2. A certain remote controlled car moves through the xy-plane by performing sequences of the following three moves:

- "$L(\theta)$", which is rotating θ degrees to the left. (Here $\theta \geq 0$ can be any real number.)
- "$R(\theta)$", which is rotating θ degrees to the right. (Here $\theta \geq 0$ can be any real number.)
- "$F(x)$", which is stepping forward x units in the direction it is facing. (Here $x \geq 0$ can be any real number.)

Assume the car starts at the origin $(0, 0)$ facing towards the positive x-axis. We want to know what points the car can reach. (In this question, "reach" means "reach and stop at", not "pass through".)

(1) Show that the car can reach the point $(-2, 2)$.
(2) Make a conjecture about what points the car can reach, and then prove your conjecture.
(3) Due to mild signal interference, the car can no longer accept instructions to make $R(\theta)$ moves (only $L(\theta)$ and $F(x)$ moves). Make a conjecture about what points the car can now reach, and then prove your conjecture.
(4) Due to severe signal interference, the car can only accept instructions to make $L(\theta)$ moves and $F(1)$ moves. Make a conjecture about what points the car can now reach, and then prove your conjecture.

Problem 3. Question 1.5.31 (all parts) from the textbook. (You may assume, as the textbook says, that $\sqrt{7}, \pi, \log_2 3$ are irrational. If you claim that any other numbers are irrational, you should prove your claims.)

Problem 4. A tutorial group of MAT102 is playing the "MAT102 Warrior" game. The TA has prepared some "MAT102 warrior" tags, and has put the k tags on the back of t-shirts of k different students. The TA will then say: "MAT102 warrior, assemble!". If a student figures out that they have a tag on their back, they will go to the blackboard, otherwise they will not; If any of the "MAT102 Warriors" have not yet come to the blackboard, the TA will ask another round, and so on. When all students with "MAT102 warrior" tags come to the blackboard, the game is over.

We take the following as granted (*axioms*):

A1. The tutorial group has only finitely many students, but always more than the number of tags k;
A2. Every student knows the total number of tags k;
A3. k is a natural number;
A4. Every student can see the back of anyone except their own back;
A5. Everyone can see the blackboard;
A6. Every student in this tutorial group is identically extremely reasonable and clever.

We would like to study this game:

(1) Using the axioms, prove that each round after the TA summoned them, either (1) all the students with "MAT102 warriors" tag will figure out they are a "MAT102 warrior", or (2) none of them figures out they are a "MAT102 warrior". This will be your *lemma*.

(2) You make the following definitions:

Definition 1:
The **first assemble number** is the number of rounds it takes for the first "MAT102 warrior" to go to the blackboard.

Definition 2:
The **last assemble number** is the number of rounds it takes for the last "MAT102 warrior" to go to the blackboard.

Using the lemma you proved before, prove that **first assemble number**=**last assemble number**. We can henceforth call it the **assemble number**, which is well defined.

(3) If $k = 1$, what is the assemble number?
(4) If $k = 2$, what is the assemble number?
(5) If $k = 100$, what is the assemble number?

Now drop the axiom

"Every student knows the total number of tags k";

(6) If $k = 1$, what is the assemble number?
(7) If $k = 2$, what is the assemble number?
(8) If $k = 100$, make a conjecture on the assemble number? (you don't need to prove it now, as we will come back to the story of "MAT102 warriors" later in the semester.)