

MAT102H5 Y - SUMMER 2020 - QUIZ 5

SUBMISSION

- **You must submit your completed Quiz on Crowdmark by 6:00pm (EDT) Tuesday August 11, 2020.**
- Late submissions will not be accepted.
- You should start uploading your quiz no later than 5:45pm.
- If you require additional space, please insert extra pages.
- You do not need to print out this quiz; you may submit clear pictures/scans of your work on lined paper, or screenshots/PDFs of your work.

ADDITIONAL INSTRUCTIONS

You must justify and support your solution to each question.

PERMITTED RESOURCES

During the quiz:

- (1) You may use any resources (course notes, textbook, videos) that have been posted to Quercus by instructors or TAs.
- (2) You may use personal notes related to official course material (from reading the textbook, participating in lectures/tutorials, completing problem sets).
- (3) Do not use personal notes related to other material (e.g. notes created by studying external websites)
- (4) Do not communicate with anyone other than the instructors.
- (5) Do not use Piazza.
- (6) Do not use any online resources other than Quercus and Crowdmark.

ACADEMIC INTEGRITY

By submitting this quiz you affirm that your submission represents entirely your own efforts. You confirm that:

- You have not copied any portion of this work.
- You have not allowed someone else in the course to copy this work.
- You understand the consequences of violating the University's academic integrity policies as outlined in the *Code of Behaviour on Academic Matters*.



PROBLEM 1 [10 POINTS]

(1) Show that for all sets A, B, C if $|A| \leq |B|$ and $|B| = |C|$, then $|A| \leq |C|$.

(2) Let $A, B_1, B_2, \dots, B_{2020}, C$ be sets such that $|A| \leq |B_1| \leq |B_2| \leq \dots \leq |B_{2020}| \leq |C|$. If $|C| = |A|$, then

$$|B_1| = |B_2| = \dots = |B_{2020}|.$$

You may use any results from class without proving them.

PROBLEM 2 [10 POINTS]

A function $f : B \rightarrow C$ is defined to be a double cover if

$$\forall c \in C, \exists b_1, b_2 \in B, \text{ such that } b_1 \neq b_2 \text{ and } f(b_1) = c \text{ and } f(b_2) = c.$$

- (1) Prove that the following function is a double cover, $f : [1, \infty) \rightarrow \mathbb{N}$ defined by $f(x)$ is “ x rounded down to the nearest integer”. For example, $f(2.99) = 2$, $f(\pi) = 3$, and $f(9) = 9$.

- (2) Prove that if $g : A \rightarrow B$ is a surjection and $f : B \rightarrow C$ is a double cover, then $g \circ f$ is a double cover.